TRINITY COLLEGE

Year 12 Mathematics Applications
Test 4 2020

Calculator Assumed
Time Series Data

STUDENT'S NAME Solutions - Martin

DATE: Friday 3rd July **TIME:** 50 minutes **MARKS**: 50

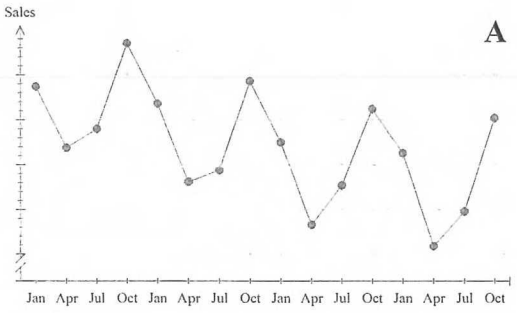
INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser
Special Items: Notes on 1 A4 page (one sided), up to 3 calculators

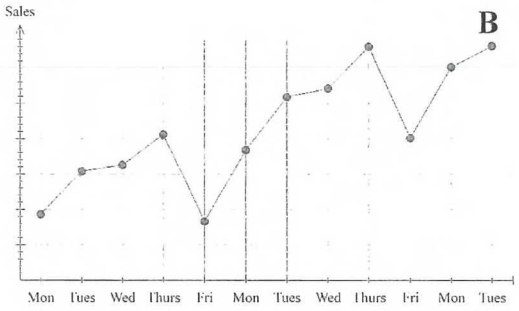
Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

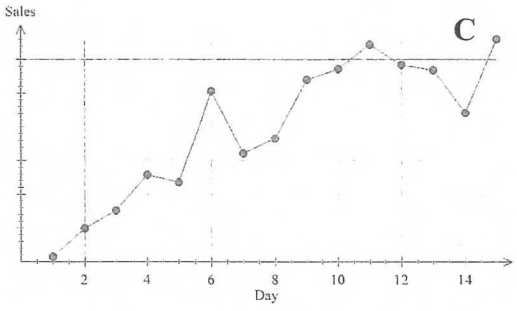
---

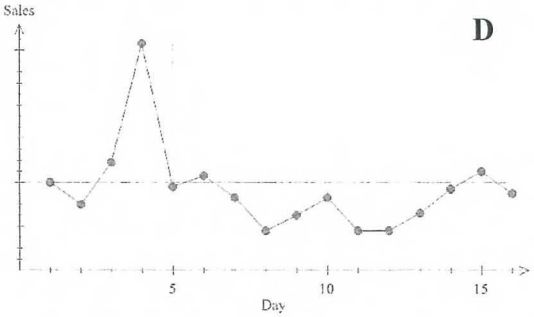
1. (7 marks)

Consider the time series plots below









(a) Describe the features of Graph A [3]

- Decreasing Trend
- 4 pt cycle
- Peaks in Oct

(b) Which graph would it be appropriate to smooth using a 5-point moving mean? [1]

B

(c) Which graph/s show an increasing trend? [2]

B/C

(d) Which graph contains an outlier? [1]

D

2. (11 marks)

A Zoo recorded the number of visitors it had in quarterly intervals from 2016 to 2019. The data is displayed in the table below

Year	(n)	Time Period	Visitors (in 1000's)	Cycle Mean	Seasonal Effect	Deseasonalised Data (D)
2016	1	Jan - Mar	78	77	101.3%	75.8
	2	Apr - Jun	72		93.5%	78.5
	3	Jul - Sept	74		**C**	77.4
	4	Oct - Dec	84		109.1%	76.6
2017	5	Jan - Mar	77	73	105.5%	74.0
	6	Apr - Jun	**A**		91.8%	74.4
	7	Jul - Sept	69		94.5%	72.5
	8	Oct - Dec	79		108.2%	72.8
2018	9	Jan - Mar	73	**B**	104.3%	69.8
	10	Apr - Jun	63		90.0%	69.2
	11	Jul - Sept	68		97.1%	70.8
	12	Oct - Dec	76		108.6%	69.9
2019	13	Jan - Mar	71	68	104.4%	68.7
	14	Apr - Jun	61		89.7%	66.5
	15	Jul - Sept	65		95.6%	**D**
	16	Oct - Dec	75		110.3%	69.0

(a) Calculate the values of A, B and C in the table. [3]

A = 67

B = 70

C = 96.1%

(b) Calculate the seasonal indices for each Quarter. [2]

Quarter	**1 (Jan – Mar)**	**2 (Apr – Jun)**	**3 (Jul – Sept)**	**4 (Oct – Dec)**
Seasonal Index	104%	91%	96%	109%

(c) Calculate the value of **D** in the table. [1]

$$\frac{65}{0.96} = 67.7$$

(d) Determine the equation of the least squares' regression line for the Deseasonalised number of visitors to the zoo. [2]

$$\hat{D} = -0.725n + 78.2625$$

(e) Predict the actual number of visitors to the zoo in the second quarter (Apr – Jun) in 2020. [3]

$$\hat{D} = -0.725(18) + 78.2625 \quad \checkmark$$

$$= 65.2125$$

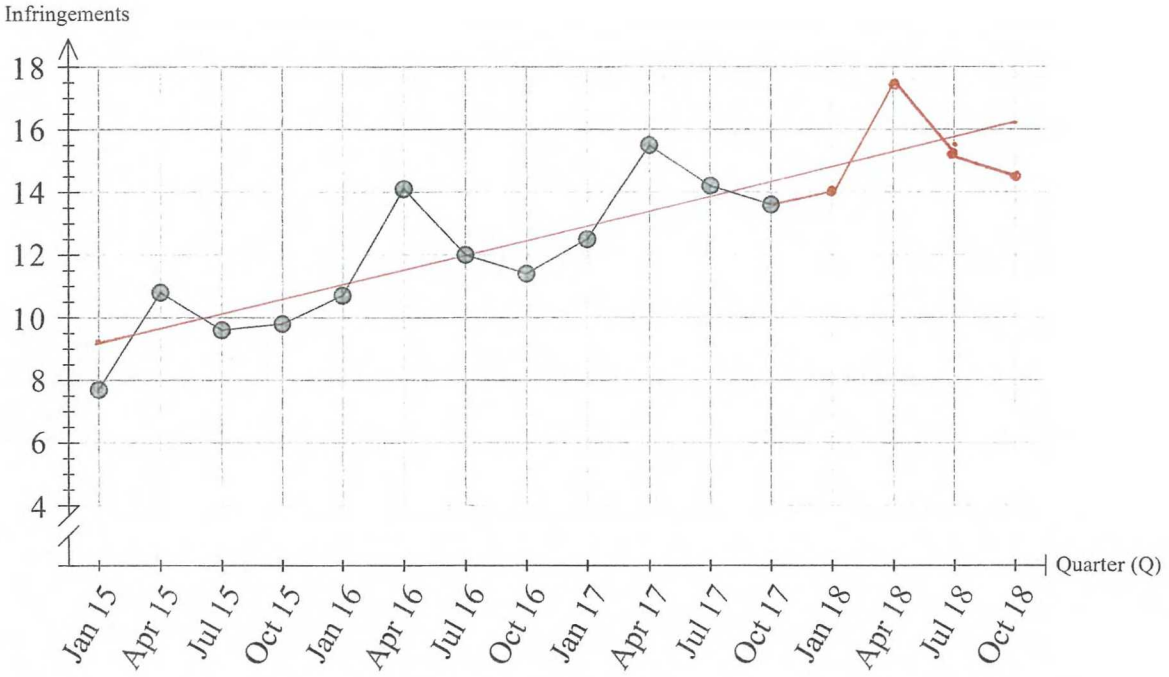
$$65.2125 \times 0.91 \quad \checkmark$$

$$= 59.343$$

$$\therefore \quad 59343 \text{ visitors} \quad \checkmark$$

3. (13 marks)

The graph below shows the quarterly speeding infringements (in 1000's) given out in Western Australia.



The data for the next four quarters are shown in the table below:

Quarter	January 2018	April 2018	July 2018	October 2018
Infringements	14 000	17 500	15 250	14 500

(a) Complete the time series plot by including this additional information. [2]

(b) The equation of the least-squares line for the above data is $I = 466.1029Q + 8735$ where $Q=1$ for January 15 and $Q=2$ for April 15 etc.

(i) Plot this line on the graph above. [2]

(ii) Describe the trend and seasonality of this data. [2]

✓ Increasing Trend

Peaks in April, each year

✓ OR

Troughs in Jan each year

(c) The 4-point centred moving average for April 2018 is 15 200. Calculate the actual value for October 2017. [2]

$$\frac{0.5x + 14000 + 17500 + 15250 + 14500 \times 0.5}{4} = 15\,200$$

$$x = 13\,600$$

(d) The seasonal indices are shown in the table below:

Quarter	Seasonal Index
January	87.5%
April	114.1%
July	100.6%
October	97.8%

(i) Complete the table above by determining the seasonal index for October. [1]

(ii) Use the seasonal index to determine the Deseasonalised number of infringements for January 2018. [2]

$$\frac{14000}{0.875} = 16\,000$$

(iii) The Deseasonalised number of infringements for April 2017 is 13 600. Determine the **actual** number of infringements for this quarter. [2]

$$13\,600 \times 1.141$$

$$= 15\,517.6$$

$$\approx 15\,518$$

4. (11 marks)

The table below shows the number of students absent from a school for each quarter for the years 2012 to 2015.

Year	(t)	Time Period	Absentees	Cycle Mean	Seasonal Effect
2012	1	Jan - Mar		47	
	2	Apr - Jun			89.8%
	3	Jul - Sept	45		
	4	Oct - Dec	58		124
2013	5	Jan - Mar	39	46	85.7%
	6	Apr - Jun	38		83.5%
	7	Jul - Sept	45		98.9%
	8	Oct - Dec	60		131.9%
2014	9	Jan - Mar	38	42	91.6%
	10	Apr - Jun	33		79.5%
	11	Jul - Sept	40		96.4%
	12	Oct - Dec	55		132.5%
2015	13	Jan - Mar	29	35	81.7%
	14	Apr - Jun	31		87.3%
	15	Jul - Sept	36		101.4%
	16	Oct - Dec	46		129.6%

For the data above, when t = 12, the least-squares regression line for Deseasonalised data against t, gives a predicted value of 39.11 and the predicted actual number of absentees is 50.65

Using this information

(a) Calculate the seasonal index for the quarter that corresponds to $t = 12$. [2]

$$\frac{50.65}{39.11} = 1.295$$

(b) Calculate the seasonal effect when t = 4. [2]

$$\frac{x + 131.9 + 132.5 + 129.6}{4} = 129.5$$

$$x = 124\ \%$$

(c) Calculate the cycle mean for 2012. [2]

$\frac{58}{x} \times 100 = 124$

$x = 47$

(d) Calculate the actual absentees for Jan – Mar of 2012. [3]

Seasonal effect when $t = 3$ $\frac{45}{47} \times 100 = 95.7\%$ ✓

season effect when $t = 1$ = 90.5 % ✓

Absentees = 0.905×47 = 43 ✓

(e) Given that the absentees for Jan – March of 2016 is 31. Estimate the total absentees for the year of 2016. [2]

$\frac{31}{87.4} \times 400$ ✓

= 142 ✓

5. (8 marks)

Consider the table of data below.

Time (t)	Sales (in \$000s)	3pt MA	4pt CMA	5pt MA
1	8.6			
2	28.6	20.87		
3	25.4	21.37	21.05	20.86
4	10.1	22.37	23.63	**C**
5	31.6	22.63	21.84	21.36
6	26.2	**A**	23.43	23.22
7	13.5	24.80	26.16	26.98
8	34.7	25.70	24.70	24.10
9	28.9	26.93	26.59	26.38
10	17.2	27.90	29.49	30.44
11	37.6	29.53	28.83	28.40
12	**D**		31.19	30.72
13	24.5	32.93	34.01	
14	40.5	33.97	32.94	32.32
15	36.9	34.43	34.04	33.80
16	25.9	34.67	**B**	36.32
17	41.2	34.73	33.91	33.42
18	37.1	34.77	34.39	34.16
19	26.0	34.57		
20	40.6			

(a) What is the purpose of calculating moving averages for time series data? [1]

Smooth data & identify underlying trend more easily.

(b) Calculate the values of A, B and C. [3]

$A = 23 \cdot 77$

$B = 35 \cdot 7$

$C = 24 \cdot 38$

(c) Which of the moving averages is the most appropriate for smoothing this data and why? [2]

3pt MA, Data appears to have a 3-pt cycle.
✓ ✓

(d) Calculate the value of D. [2]

$$\frac{17 \cdot 2 + 37 \cdot 6 + D}{3} = 29 \cdot 53$$

$D = 33 \cdot 8$